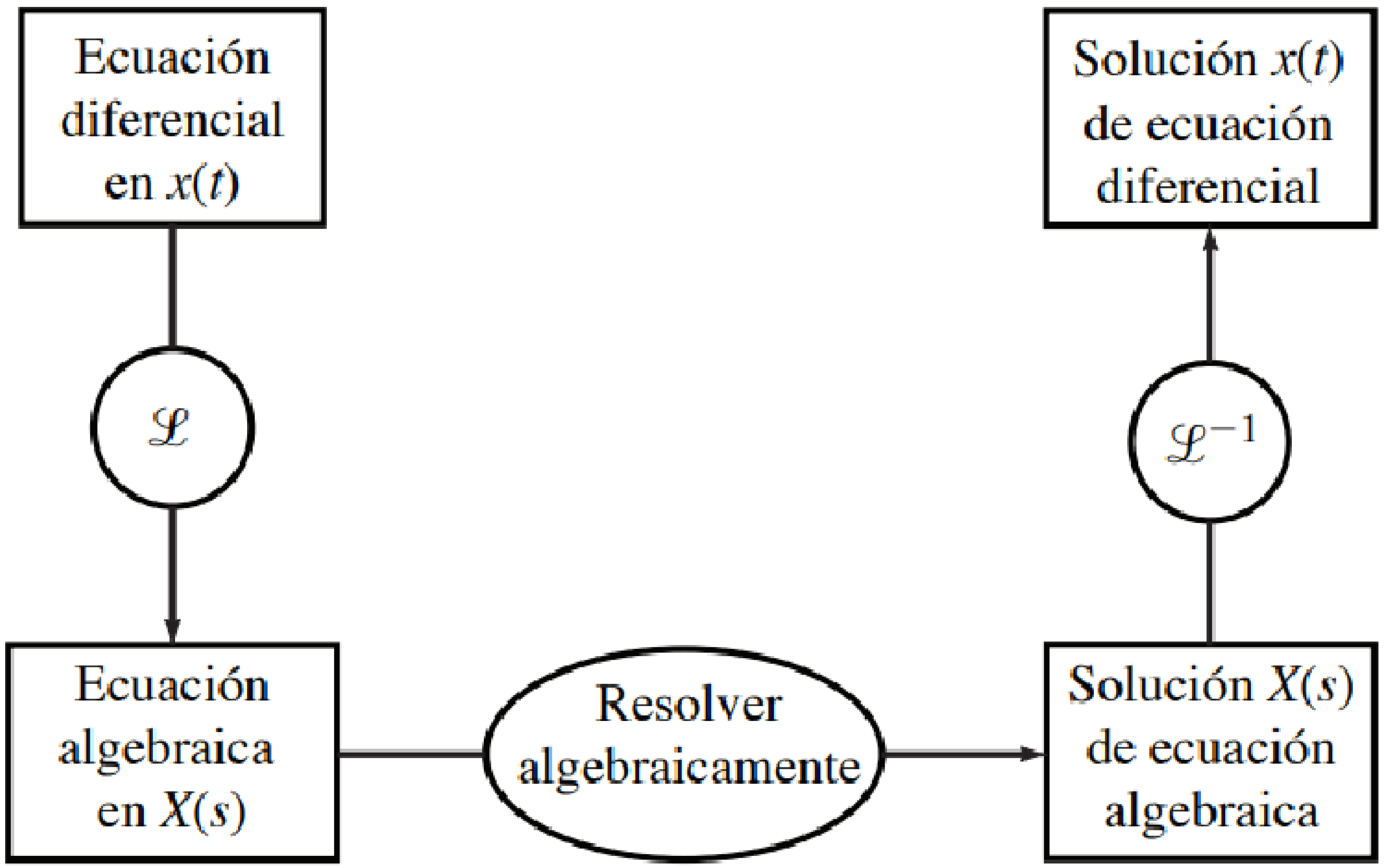

Ecuación diferencial en $x(t)$
$\mathscr{L}$
Ecuación algebraica en $X(s)$
Resolver algebraicamente
Solución $X(s)$ de ecuación algebraica
$\mathscr{L}^{-1}$
Solución $x(t)$ de ecuación diferencial

(iii) $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+2s-3}\right\}$

$s^2+2s-3 \xrightarrow{1)}$ se puede factorizar — Fracciones parciales

$\xrightarrow{2)}$ si no se puede — Completar cuadrados

¿1? $\quad s^2+2s-3=(s+3)(s-1)$

¿?

$$\frac{s}{\boxed{s^2+2s-3}}=\frac{s}{(s+3)(s-1)}=\frac{A}{s+3}+\frac{B}{s-1}$$

$$\frac{s}{(s+3)(s-1)} = \frac{A}{s+3} + \frac{B}{s-1} \quad \Big/ \cdot (s+3)(s-1)$$

$$s = A(s-1) + B(s+3)$$

$s = 1$: $1 = A \cdot 0 + B \cdot 4 \Rightarrow 1 = 4B \Rightarrow \boxed{B = \frac{1}{4}}$

$s = -3$: $-3 = A \cdot (-4) + B \cdot 0 \Rightarrow -3 = -4A \Rightarrow$ $\boxed{A = \cancel{+}\frac{3}{4}}$

$$\therefore \boxed{\frac{s}{(s+3)(s-1)} = \frac{3/4}{s+3} + \frac{1/4}{s-1}}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\} = \mathcal{L}^{-1}\left\{\frac{3}{4}\cdot\frac{1}{s+3} + \frac{1}{4}\frac{1}{s-1}\right\}$$

$$= \frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

$$= \frac{3}{4}e^{-3t} + \frac{1}{4}e^{t}$$

o 3ro caso:

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+8}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{\boxed{s^2+4s+4}-4+8}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{(s+2)-2}{(s+2)^2+4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+4}\right\} - \frac{2}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s+2)^2+2^2}\right\}$$

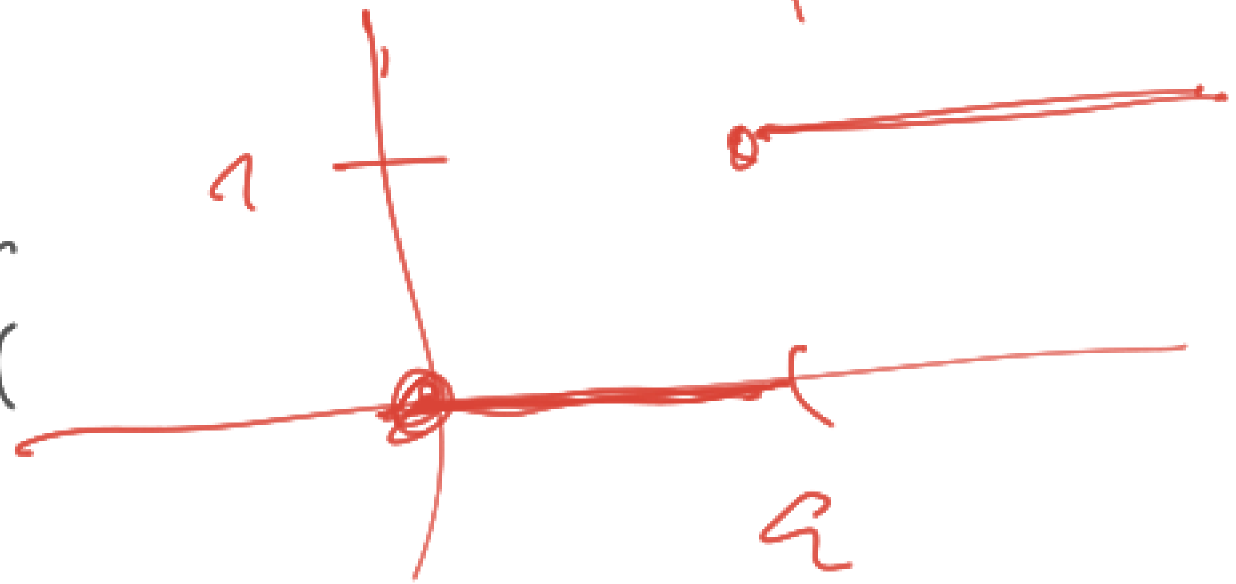
$$= e^{-2t}\cos(2t) - \frac{1}{2}e^{-2t}\operatorname{Sen}(2t)$$

(iv) $\mathcal{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+1}\right\} =$

$$\mathcal{L}\{f(t-a)\,\underline{u(t-a)}\} = e^{-as}F(s)$$

$$\mathcal{L}\{\underbrace{f(t-a)\,u(t-a)}_{\text{buscamos}}\} = \underbrace{e^{-\pi s}\,\frac{1}{s^2+1}}_{\text{Tenemos}}$$

$$u(t-a) = \begin{cases} 1, & t \geq a \\ 0, & t < a \end{cases}$$




$$\underline{\underline{f(t-\pi)}}\,u(t-\pi) \qquad F(s) = \frac{1}{s^2+1}$$

$$\downarrow$$

$$\underbrace{\text{sen}(t-\pi)\,u(t-\pi)}_{f(t)} \qquad f(t) = \underline{\underline{\text{sen}(t)}}$$

$$\text{sen}(t-\pi) = -\text{sen}(\pi-t) = -\text{sen}\,t$$

$$\boxed{f(t) = -\text{sen}\,t\; u(t-\pi)}$$

b) $y'' - 6y' + 9y = t^2 e^{3t}$, $y(0) = 2$, $y'(0) = 17$ **Sol.**: $y(t) = e^{3t}(2 + 11t + \frac{1}{12}t^4)$

$\mathcal{L}\{y'' - 6y' + 9y\} = \mathcal{L}\{t^2 e^{3t}\}$

$\mathcal{L}\{y''\} - 6\mathcal{L}\{y'\} + 9\mathcal{L}\{y\} = \dfrac{2}{(s-3)^3}$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a) = \mathcal{L}\{f(t)\}(s-a) = \mathcal{L}\{f(t)\}_{s \to s-a}$$

$\mathcal{L}\{e^{3t} \cdot t^2\} =$

$f(t)$

$\mathcal{L}\{t^2\} \overset{F1}{=} \dfrac{2!}{s^3} = \dfrac{2}{s^3}$

$$\mathcal{L}\{y''\} - 6\mathcal{L}\{y'\} + 9\mathcal{L}\{y\} = \frac{2}{(s-3)^3}$$

$$s^2\mathcal{L}\{y\} - s\,y(0) - y'(0) - 6\left(s\mathcal{L}(y) - y(0)\right) + 9\mathcal{L}(y) = \frac{2}{(s-3)^2}$$

$y(0) \to 2$, $y'(0) \to 17$, $y(0) \to 2$

$$s^2 Y(s) - 2s - 17 - 6s\,Y(s) + 12 + 9\,Y(s) = \frac{2}{(s-3)^2}$$

$$Y(s)\left(s^2 - 6s + 9\right) = \frac{2}{(s-3)^2} + 2s + 5$$

$$Y(s)\,(s-3)^2 = \frac{2}{(s-3)^2} + 2s + 5$$

$$Y(s) = \frac{2}{(s-3)^4} + \frac{2s+5}{(s-3)^2}$$

$y(t)$?

$$Y(s) = \frac{2}{(s-3)^4} + \frac{2s+5}{(s-3)^2} \quad \mathcal{L}^{-1}$$

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left\{\frac{2}{(s-3)^4} + \frac{2s+5}{(s-3)^2}\right\}$$

$$\to Y(t) = \frac{1}{3}e^{3t}\left(t^2 + 33t + 6\right)$$

¡TAREA!