Método de las TdL para resolver SED

$3x' - y = 2t,\ x' + y' - y = 0;\ x(0) = y(0) = 0$

(1) $3x' - y = 2t$

(2) $x' + y' - y = 0$

$x(0)=0,\ y(0)=0$ ⟵ Cond. iniciales

$x = x(t)$
$y = y(t)$

$\mathcal{L}\{f'\} = s\,\mathcal{L}\{f\} - f(0)$

P1) Aplicar TdL a cada ecuación.

(1) $\mathcal{L}\{3x' - y\} = \mathcal{L}\{2t\}$

$\mathcal{L}\{3x'\} - \mathcal{L}\{y\} = 2\mathcal{L}\{t\}$

$3\boxed{\mathcal{L}\{x'\}} - \mathcal{L}\{y\} = 2\boxed{\mathcal{L}\{t\}}$

$3(sX(s) - x(0)) - Y(s) = 2\cdot\frac{1}{s^2}$

$3sX(s) - 0 - Y(s) = \frac{2}{s^2}$

1': $3sX(s) - Y(s) = \frac{2}{s^2}$

(2) $\mathcal{L}\{x' + y' - y\} = \mathcal{L}\{0\}$

$\mathcal{L}\{x'\} + \mathcal{L}\{y'\} - \mathcal{L}\{y\} = 0$

$sX(s) - \cancel{x(0)} + sY(s) - \cancel{y(0)}^{0} - Y(s) = 0$

2': $sX(s) + sY(s) - Y(s) = 0$

$$3s\,X(s) - Y(s) = \frac{2}{s^2}$$
$$s\,X(s) + (s-1)\,Y(s) = 0$$

Vamos a usar la Regla de Cramer:

$$\Delta = \begin{vmatrix} 3s & -1 \\ s & s-1 \end{vmatrix} = 3s(s-1) + s = 3s^2 - 3s + s = \boxed{3s^2 - 2s} \ *$$

$$\Delta X(s) = \begin{vmatrix} \frac{2}{s^2} & -1 \\ 0 & s-1 \end{vmatrix} = \frac{2}{s^2}(s-1) = \boxed{\frac{2}{s} - \frac{2}{s^2}} \leftarrow$$

$$\Delta Y(s) = \begin{vmatrix} 3s & \frac{2}{s^2} \\ s & 0 \end{vmatrix} = -\frac{2}{s^2}\cdot s = \boxed{-\frac{2}{s}}$$

$$X(s) = \frac{\Delta X(s)}{\Delta} = \frac{\frac{2}{s} - \frac{2}{s^2}}{3s^2 - 2s} = \frac{\frac{2s-2}{s^2}}{s(3s-2)}$$

$$\boxed{X(s) = \frac{2s-2}{s^3(3s-2)}}$$

$$Y(s) = \frac{\Delta Y(s)}{\Delta} = \frac{-2}{s \cdot s(3s-2)}$$

$$\boxed{Y(s) = \frac{-2}{s^2(3s-2)}}$$

$$\boxed{X(s)=\frac{2s-2}{s^3(3s-2)}}$$

$$X(s)=\frac{-9}{4(3s-2)}+\frac{1}{s^3}+\frac{1}{2s^2}+\frac{3}{4s}\quad\Big|\ \mathcal{L}^{-1}$$

$$\underbrace{\mathcal{L}^{-1}\{X(s)\}}=-\frac{9}{4}\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{3s-2}\right\}}+\underbrace{\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\}}+\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}+\frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$X(t)=-\frac{9}{4\cdot 3}\mathcal{L}^{-1}\left\{\frac{1}{s-\frac{2}{3}}\right\}+\frac{1}{2}t^2+\frac{1}{2}\cdot t+\frac{3}{4}\cdot 1$$

$$\boxed{X(t)=-\frac{3}{4}e^{\frac{2t}{3}}+\frac{1}{2}t^2+\frac{1}{2}t+\frac{3}{4}}$$

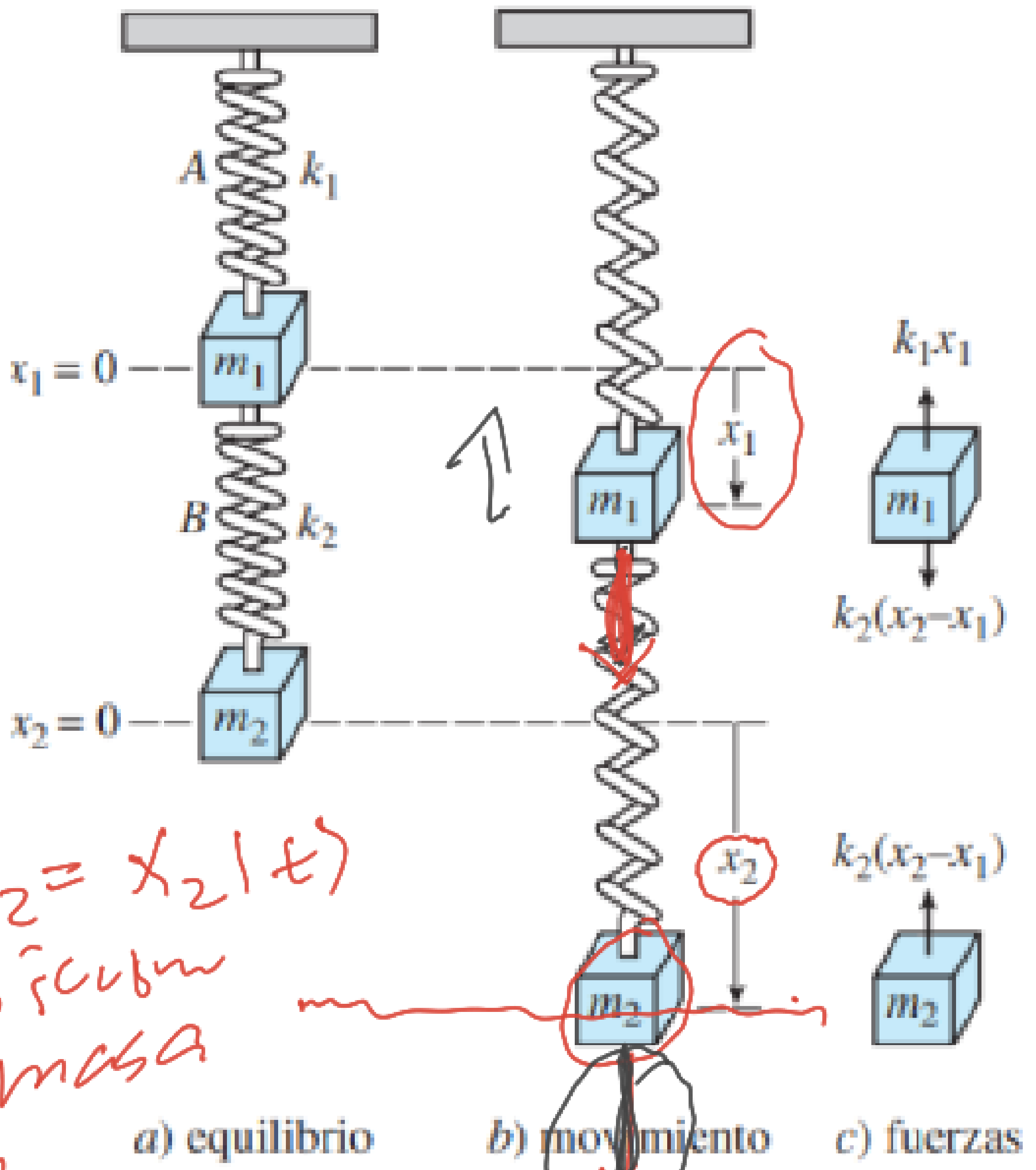
$$\boxed{Y(s) = \frac{-2}{s^2(3s-2)}}$$

$$Y(s) = \frac{-9}{2(3s-2)} + \frac{1}{s^2} + \frac{3}{2}\cdot\frac{1}{s} \quad \Big/ \mathcal{L}^{-1}$$

$$\mathcal{L}^{-1}\{Y(s)\} = -\frac{9}{2}\mathcal{L}^{-1}\left\{\frac{1}{3s-2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \frac{3}{2}\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}}$$

$$y(t) = -\frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{s-\frac{2}{3}}\right\} + t + \frac{3}{2}\cdot 1$$

$$\boxed{y(t) = -\frac{3}{2}e^{\frac{2}{3}t} + t + \frac{3}{2}} \checkmark$$



$X_2 = X_2(t)$

posición la masa m_2

m_1

$m \cdot a = \sum F_i$

$F_1 = -k_1 \cdot x_1$

m_1

$F_2 = k_2 \underbrace{(x_2 - x_1)}$

$$\boxed{m_1 \cdot x_1'' = -k_1 x_1 + k_2(x_2 - x_1)}$$

m_2

$m_2 a = -k_2(x_2 - x_1)$

$m_2 x_2'' = -k_2(x_2 - x_1)$

Use la transformada de Laplace para resolver

$$
\begin{aligned}
(1)\quad & x_1'' + 10x_1 - 4x_2 = 0 \\
(2)\quad & -4x_1 + x_2'' + 4x_2 = 0
\end{aligned}
\qquad (2)
$$

sujeto a $x_1(0) = 0$, $x_1'(0) = 1$, $x_2(0) = 0$, $x_2'(0) = -1$. Éste es el sistema (1) con $k_1 = 6$, $k_2 = 4$, $m_1 = 1$ y $m_2 = 1$.

$$m_1 \frac{d^2x_1}{dt^2} = -k_1x_1 + k_2(x_2 - x_1).$$

$$m_2 \frac{d^2x_2}{dt^2} = -k_2(x_2 - x_1).$$

Se aplica TdL a cada ec. del sistema:

$$(1)\quad \mathcal{L}\{x_1''\} + 10\,\mathcal{L}\{x_1\} - 4\,\mathcal{L}\{x_2\} = 0$$

$$s^2X_1(s) - s\cancel{x_1(0)} - x_1'(0) + 10X_1(s) - 4X_2(s) = 0$$

$$s^2X_1(s) - 1 + 10X_1(s) - 4X_2(s) = 0$$

$$\boxed{1':\quad (s^2+10)X_1(s) - 4X_2(s) = 1}$$

$$(2)\quad -4X_1(s) + s^2X_2(s) - s\cancel{x_2(0)} - x_2'(0) + 4X_2(s) = 0$$

$$\boxed{2':\quad -4X_1(s) + (s^2+4)X_2(s) = -1}$$

$$(s^2+10)\,X_1(s) - 4\,X_2(s) = 1$$
$$-4\,X_1(s) + (s^2+4)\,X_2(s) = -1$$

Usamos Cramer:

$$\Delta = \begin{vmatrix} s^2+10 & -4 \\ -4 & s^2+4 \end{vmatrix} = (s^2+10)(s^2+4) - 16$$
$$= s^4 + 14s^2 + 40 - 16$$
$$= s^4 + 14s^2 + 24$$
$$= (s^2+2)(s^2+12)$$

$$\Delta X_1(s) = \begin{vmatrix} 1 & -4 \\ -1 & s^2+4 \end{vmatrix} = s^2 + 4 - 4 = s^2$$

$$\Delta X_2(s) = \begin{vmatrix} s^2+10 & 1 \\ -4 & -1 \end{vmatrix} = -s^2 - 10 + 4$$
$$= -s^2 - 6$$

$$X_1(s) = \frac{s^2}{(s^2+2)(s^2+12)}$$

$$= \frac{6}{5}\,\frac{1}{s^2+12} - \frac{1}{5}\,\frac{1}{s^2+2}$$

$$\Downarrow \mathcal{L}^{-1}$$

$$x_1(t) = \frac{6}{5\sqrt{12}} \cdot \frac{\sqrt{12}}{s^2+(\sqrt{12})^2} - \frac{1}{5\sqrt{2}}\,\frac{\sqrt{2}}{s^2+\sqrt{2}}$$

$$\boxed{x_1(t) = \frac{6}{5\sqrt{12}}\sin(\sqrt{12}\,t) - \frac{1}{5\sqrt{2}}\sin(\sqrt{2}\,t)}$$

$$X_2(s) = \frac{-s^2-6}{(s^2+2)(s^2+12)}$$

$$= -\frac{3}{5}\,\frac{1}{s^2+12} + \frac{2}{5}\,\frac{1}{s^2+2}$$

$$\Downarrow \mathcal{L}^{-1}$$

$$x_2(s) = -\frac{3}{5\sqrt{12}} \cdot \frac{\sqrt{12}}{s^2+(\sqrt{12})^2} + \frac{2}{2\sqrt{2}}\,\frac{\sqrt{2}}{s^2+\sqrt{2}}$$

$$\boxed{x_2(t) = -\frac{3}{5\sqrt{12}} \cdot \sin(\sqrt{12}\,t) + \frac{2}{2\sqrt{2}}\sin(\sqrt{2}\,t)}$$